Support for the SME Supporting Factor - Multi-country empirical evidence on systematic risk factor for SME loans[1][2]

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Abstract

Using a unique and comprehensive data set on the two largest economies of the Eurozone – France and Germany – this paper first proceeds to a computation of the Gordy formula relaxing the ad hoc size-dependent constraints of the Basel formulas. Our study contributes to Article 501 of the Capital Requirements Regulation (CRR) requesting analysis the consistency of own funds requirements with the riskiness of SMEs. In both the French and the German sample, results suggest that the relative differences between the capital requirements for large corporates and those for SMEs (in other words the capital relief for SMEs) are lower in the Basel III framework than implied by empirically estimated asset correlations. Results show that the SME Supporting Factor in the CRR/CRDIV is able to compensate the difference between estimated and CRR/CRDIV capital requirements for loans in the corporate portfolio.

Keywords: SME Supporting Factor, Asset correlation, Basel III, Minimum Capital requirements, Asymptotic Single Risk factor Model, SME finance

JEL-Classification: G21, G33, C13

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1. Motivation

In the Basel II / III framework, banks capital requirements for corporate credit risk rely on the standard asymptotic single risk factor model (ASRF) as proposed by Gordy (2003). In this framework, provided that two assumptions are satisfied - portfolio are granular, and there exists one single systematic risk factor - required capital is defined by using three main risk parameters. Two of them are defined at the exposure level: the probability of default and the loss given default. The third one is defined at the portfolio's level: that is the correlation to the systematic factor. However, the calibration of the requirements by the Basel Committee departs from this theoretical model. In particular, the correlation parameter is fixed by the regulatory formulas as a function of the firm's size and its probability of default and it is invariant with the characteristics of the real credit portfolios. Thus, one issue comes from the fact that capital requirements could not necessarily reflect the banks' portfolio effective credit risk. This issue of the consistency of own funds requirements with the riskiness of SMEs is the main issue addressed in this paper.

This issue is of a particular importance for small and medium-sized enterprises (SMEs) financing, which is a growing concern in Europe. As illustrated by recent contributions to the empirical literature[3], banks' lending decisions are sensitive to capital requirements. This paper assesses whether the differences implied by the Basel formulas between SME and large corporates capital requirement are empirically justified in the data from a management risk perspective. This question became a salient issue from a policy perspective. One concern was that Basel III, by imposing higher capital requirements (from 8% to at least 10.5% due to the conservation buffer), might also affect unfairly capital requirements for credit exposures to SMEs given that these firms did not cause the recent financial crisis. The CRR/CRDIV (Art. 501) has thus introduced a deduction in the capital requirements for exposures to SMEs, which will be reduced through the application of a "SME Supporting Factor" equal to 0.7619 (8%/10.5%). This reduction was subject to a review by the European Banking Authority (EBA) which might lead to a potential revision of the SME Supporting Factor. Thus, this paper also addresses the related issue of the adequacy of the Supporting Factor.

[3] See for instance Behn et al. (2015) or Fraisse et al. (2013) respectively in the case of Germany and France.

This paper relies on two unique and comprehensive data sets which were built to cover a significant part of the SME and large corporate sector in France and Germany, the two largest economies of the Eurozone. For Germany, data from more than 1,500 banks in Germany were collected. The national credit register and the Banque de France (BDF) rating system were used for France. Given the size of the two samples, the length of the time series and the application of a fully consistent methodology, this paper significantly improves results in the previous literature in this area. As noticed previously, the asset correlation is a key driver of portfolios' credit risk. Using the information provided by the two databases, this paper proceeds to the computation of the asset correlation using Gordy formula while relaxing the ad hoc constraints of the Basel formulas and it compares the estimated capital requirements with the regulatory requirements. Moreover, our paper addresses the consistency of own funds requirements by assessing firm size as a driver of systematic credit risk in loans to SMEs and compares the size of this effect with the capital relief granted to SME lending relative to large corporates in the regulatory minimum capital requirements of Basel III and the CRR/CRDIV (including the capital relief from the application of the SME Supporting Factor). Indeed, an evaluation of regulatory capital requirements should distinguish between the level of capital and the relative difference against other asset classes. In the development of Basel II, the second aspect – often referred to as relative calibration – was addressed first. It ensures that banks ceteris paribus have to hold more (less) capital for a more (less) risky asset, while the level calibration aims at determining the overall level of capital requirements. This study focusses on the relative calibration of the risk weights. Therefore, we use large corporates (with a turnover of more than € 50 mln) as a benchmark, our research question being on the existence of a regulatory distortion between small and large firms. Our relative approach is also motivated by the fact that the Basel Committee on Banking Supervision (BCBS) has spent substantial effort on calibrating the large corporates portfolios due to their immense economic importance. In this paper, we consider several size classes inside the business population, and for each size class we therefore compare the relative difference in capital requirements for this size class of SMEs relative to the benchmark. In fact, we compare both (1) capital requirements based on estimated asset correlations and (2) the current IRBA capital requirements. Comparing these two relative differences can provide useful information for an evaluation of the capital relief for SMEs granted in Basel III and the

effectiveness of the SME Supporting Factor (SF) of the CRR/CRDIV[4]. If the relative differences of the capital requirements is larger for the capital requirements based on empirically estimated asset correlations than for the IRBA regulatory capital requirements, there is potential for a capital relief for SME loans. The SME SF effectively accounts for the difference in riskiness, if such a gap is reduced. This framework is applied in the same manner to a comparison with the revised standardized approach (RSA).

In the measurement of capital requirements, as mentioned previously, the asset correlation plays a major role. Accordingly, for the analysis of the consistency of own funds requirements, it is important to separate a potentially higher firm-specific (idiosyncratic) risk of SMEs – that is typically reflected in higher default probabilities – from a potentially lower systematic risk of SMEs. Since capital requirements in the ASRF model refer by construction to systematic risk, lower asset correlations (and therefore lower systematic risk) for SMEs as compared to large firms would ceteris paribus also suggest lower capital requirements for SMEs. However, in the Basel II / III framework, the capital requirements for an SME loan depend exclusively on the probability of default (PD) and the asset correlation is also defined as a function of the PD. As a consequence, higher capital requirements for SMEs may well be associated with lower systematic risk because SMEs have on average significantly higher default probabilities, i.e. higher firm-specific risk, than large firms.

In this paper, asset correlations (and the PDs) are estimated from historical time series default rates. To this aim, we choose to use the Generalized Linear Mixed Model (GLMM) single factor-estimator of Frey and McNeil (2003) in which the systematic portfolio risk is specified in terms of observable factors (fixed effects) and unobservable factors (random effects). In our application, the rating information is treated as a fixed effect, while the latent systematic risk factor corresponds to the random effect that is estimated. This estimation technique is relatively robust against low populations of rating classes and allows obtaining one single asset correlation (AC) estimate per size class. As robustness check, the Maximum Likelihood (ML)-estimator of Gordy and Heitfield (2010) is employed which can be used to estimate asset correlations and PDs for each rating/size bucket.

[4] Note that the SME SF might be alternatively viewed as a tool for supporting credit supply to the SMEs and/or as a tool for taking into account the underlying lower risk of the SMEs at the portfolio level. Our paper assesses the effectiveness of the SME Supporting Factor with respect to this second view.

Our empirical results confirm previous findings that asset correlations increase with firm size conditional on the rating category. More precisely, large corporates face a considerably higher systematic risk than SMEs and a structural difference between loans to large corporates and SME loans is identified. This result is consistent for France and Germany. Furthermore, the empirical results suggest that the relative differences between the capital requirements for large corporates and those for SMEs (in other words the capital relief for SMEs) are lower in both countries under the Basel III framework than implied by our empirically estimated asset correlations referring to the corporate portfolio under the IRBA and the RSA. These results can be transferred to the current regulatory capital requirements under the CRR/CRDIV. Our results reveal for France and Germany that under CRR/CRDIV the SME Supporting Factor is consistent with the lower systematic risk of SMEs for all exposure classes in RSA, and for corporate SMEs in the IRBA. However, for retail loans in IRBA, the capital reductions associated with the SME SF lead to relative capital requirements that are lower than those suggested by the systematic risk. As a result, after the application of the SME SF the relative regulatory risk weights are in line with the empirical ones in IRBA Corporate exposure class and SA. Furthermore, the study does not find empirical evidence supporting the limit of € 1.5 mln for the amount owed, which is currently used for the application of the SME SF in accordance with Article 501 CRR. Moreover, since the regulatory minimum capital requirements are internationally harmonized today, a key contribution of this paper lies in its international coverage. The two country samples we have used stand apart due to their comprehensive coverage of the particularly rich and well developed credit market for SMEs, the availability of banks' internal ratings, and the capture of the recent financial crisis in the time series. All in all, our results show remarkable large similarities of the credit risk structure in SME loans portfolios in the two countries.

The remainder of the paper is as follows: Section 2 presents the relation of our paper to the literature. Section 3 describes the data sample of the two countries. Section 4 presents the methodology and Section 5 the empirical results given by the GLMM estimator. The paper concludes with a summary of the key results.

2. Relation to the Literature

There exists a by now well-established strand of empirical work on the systematic risk in SME loans. Although its findings on the level of asset correlations in the ASRF model vary substantially, overall they tend to indicate lower rather than higher asset correlations compared to the values used in the IRBA capital requirements. A comprehensive overview of asset correlation studies can be found in Berg, Gehra and Kunisch (2011), and Düllmann and Koziol (2013). In general, asset correlations can be computed in two different ways, or rather by using different types of data sources. These two strands of the empirical literature find quite different results in terms of the level of asset correlations. The first possibility is the use of historical default rates.[5] These studies include Roesch (2003), Dietsch and Petey (2004), Düllmann and Scheule (2006), Palombini (2009), Haddad (2013), Bams, Pisan, and Wolff (2014) and Düllmann and Koziol (2014). These authors generally estimate lower values than those used in the IRBA. Finally, another common method is the estimation of asset correlations based on equity prices, which cannot truly form a comprehensive data set for SME loans. In the second strand, Düllmann, Kunisch, and Kuell (2010) have shown that asset correlation estimates based on equity prices tend to be somewhat higher than those based on default rates. Studies by, for instance, Hahnenstein (2004), Lopez (2004) or Chernih, Henrard and Vanduffel (2010) are in line with their results.

Several studies assess the dependence of asset correlations on size and creditor quality (i.e., rating) which is also incorporated in the Basel II/III IRB corporate risk-weights formula. Table A.1 in Appendix A provides a comprehensive overview of the existing empirical studies on the relationship between asset correlations and firm size. Lopez (2004) finds a positive size dependence of asset correlations estimated from equity prices for multiple regions (World, Japan, US, Europe). A study in this respect is Düllmann and Scheule (2006), which is based on default rates. Using Deutsche Bundesbank data they construct a time series of default rates of German firms for the years 1991 to 2000. The objective of the paper is to estimate the asset correlation of German corporate borrowers and its dependency on the firm size. For this purpose, both the Asymptotic Methods-of-Moments (AMM) and ML-estimators

[5] As default events are scarce, asset correlations estimates based on default rates are sometimes supplemented by using credit rating transition data. Examples for this approach can be found in van Landschoot (2007) and Kalkbrener and Onwunta (2009). Studies that rely on the joint direction of rating changes to estimate asset correlations include Fu et al. (2004), Akhavein, Kocagil and Neugebauer (2005) and Cassart, Castro, Langendries and Alderweireld (2007).

are used. Under both estimation methods and for all considered rating classes, asset correlation increases with firm size. In contrast, Dietsch and Petey (2004) find that for French and German SMEs "asset correlations decrease significantly on average with the SME size", while a comparison between SMEs and large corporates points towards higher asset correlations for large corporates as compared to SMEs.

In recent years the analysis of firm size as a driver of asset correlations has been extended to further regions (Japan, US, UK, Italy and Canada) and more refined data sets (e.g. Düllmann and Koziol, 2014, and Dietsch and Fraisse, 2013). The majority of studies suggest a positive relationship between asset correlations and firm size. In an empirical study of default data for Japanese companies Hashimoto (2009) shows that asset correlation varies with industry, size, credit rating and region. When grouped by size, the results exhibit higher asset correlations for large and medium-sized companies (about 4.5%) and lower asset correlations for small companies (about 1.5%). Gabbi and Vozzella (2013) use balance sheet data for small and medium-sized Italian firms for 1994 to 2008 to estimate confidence sets for asset correlations. Their results for different size clusters of small firms suggest a J-shaped relationship between asset correlation and company size, i.e. for the smallest companies, asset correlations and size are negatively interconnected, while for medium companies, the relationship shows a positive pattern. Bams, Pisa and Wolf (2015) use a multi-factor model to estimate asset correlations for a data set of US SMEs. They find that asset correlations of SMEs are negatively related to credit quality and tend to be much lower than their asset correlation estimates for corporates (based on publicly available S&P data). For the UK, Lee et al. (2013) also find a positive relationship between firm size and asset correlations estimated from asset prices (Datastream).

Our study extends Düllmann and Koziol (2014), who estimate asset correlations from a time series of default rates of SME and large corporates bank lending (2005-2011) by using the ML estimator of Gordy and Heitfield (2010). They use their asset correlation estimates for various size classes for a comparison with the minimum capital requirements in Basel II and find that the relative differences between the capital requirements for large corporates and those for SMEs (in other words, the capital relief for SMEs in Basel II) are in two cases lower in the current regulatory framework than implied by the empirically estimated asset correlations: (1) In the IRB Approach this difference amounts to up to 24 percentage points on

average across rating categories. This concerns only SME loans in the corporate portfolio. (2) This effect is considerably stronger for all loans assigned to the SME portfolio in the RSA.

Several studies have considered the size dependence of asset correlations by extending the ASRF-framework to capture additional factors. Dietsch and Petey (2007) again focus on French SMEs using a database of French SME rating and defaults provided by COFACE Services – a large French credit insurance company – for the time period of 1995 to 2005. The method chosen for estimation purposes consists of a GLMM multi-factor framework taking into account sector, location, or size specific factors in addition to a single systematic factor. Their results suggest that higher default rates do not imply higher asset correlation, but that asset correlations increase with firm size. Applying a multifactor extension of the ASRF framework to a French business loans sample, Dietsch and Fraisse (2013) recently showed that regulatory IRB capital requirement could overestimate the business cycle effect and underestimate diversification benefits when adding size or industry risk factors in the credit risk parameters estimation.

There is almost no evidence on the impact of exposure on the level of asset correlation. A notable exemption is Haddad (2013) who for a portfolio of Canadian high-risk SME loans fails to find a clear pattern for the behavior of asset correlations in relation to credit exposure. Thus this paper is one of the first to explore the dependence of asset correlation on outstanding obligos.

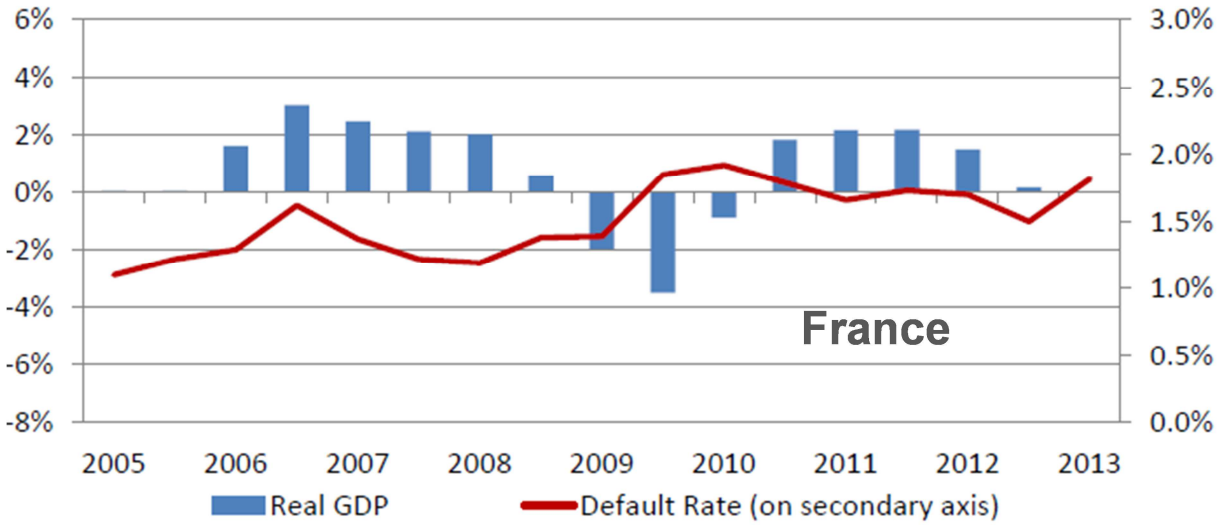
3. Data Samples

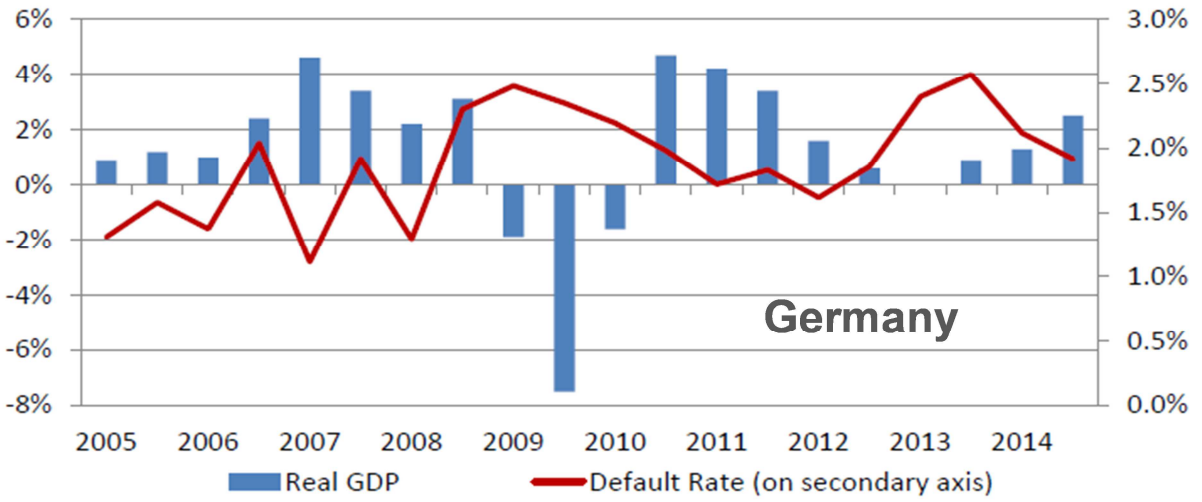
The study applies a unique data set of SME lending for France and Germany. The two samples cover a significant proportion of loans to SMEs, as well as to large corporates in the respective countries. By using a unique and comprehensive data set on the two largest economies of the Eurozone – France and Germany – our study captures a significant part of lending towards SMEs and large corporates in the European Union. It thus also improves

previous studies in terms of representativeness, as it exploits the maximum of national supervisory data that fulfills the requirements of the chosen study design.[6]

We exploit time series of default data to estimate the asset correlation. As systematic risk is driven by the evolution of the credit cycle over time, the time series should capture at least one full economic cycle. The German database covers observations from 2005 to 2014 (20 observations), whereas French data range from 2005 to 2013 (18 observations). As graphically depicted in Figure 1 each of the samples for the two countries encompasses both periods of economic growth and decline, including the financial crisis.

Figure 1: Default Rates over time (all size and rating classes) and change in real GDP





[6] Attempts to uncover further supervisory data for additional countries via an EBA data request have not been fruitful, as most available data sources lacked the required properties in terms of size, length or quality.

Figure 1 displays semi-annual changes in real GDP (left-hand axis) and the total default rate (right-hand axis) expressed as a percentage for the years 2005 to 2014 for France and Germany.

Following the specifics of each of the national data sets, different definitions of default are used.[7] The German data are based on the Basel II/III definition.[8] This definition is not only wider than the insolvency legal criterion that has often been used in previous studies; it is also consistent with the aim of our study to assess the appropriateness of the size dependence of asset correlations incorporated in the regulatory capital requirements of Basel II/III.[9] In the French database, a combination of judiciary definition and banking definition of default is used. Even if the number of defaults is expected to be higher on average when using the Basel II/III definition of default and, consequently, the results are expected to be more robust to small sample noise in buckets of low default rates, works in progress tend to show that the difference in the definition does not impact the measures of credit risk too strongly. What is more important for the robustness of asset correlation estimates is the fact that the default definitions in both countries are consistent over time. Thus, any bias resulting from a structural break in the definition of default can be avoided.

Our analysis is based on the widely known ASRF model of Gordy (2003), which is also the foundation of the IRB risk weight functions for credit exposures in the banking book. The IRB risk weights for corporate SMEs are driven by the PD and the firm size. It is therefore important to capture not only the size dependence but also the effect of diverging credit quality. This is done by clustering the two samples into size buckets and into rating buckets.

The French sample contains the population of French firms which fulfill four conditions (Table 1): i) they have exposures in the French Credit Register, ii) the BDF rating department gives them a rating (including default grades), iii) they obtain loans from at least one large banking group operating in the French loans to businesses market, and iv) their annual

[7] Any aggregation of the two data sets is therefore not applicable.
[8] "A default is considered to have occurred with regard to a particular obligor when either or both of the two following events have taken place. (1) The bank considers that the obligor is unlikely to pay its credit obligations to the banking group in full, without recourse by the bank to actions such as realising security (if held). (2) The obligor is past due more than 90 days on any material credit obligation to the banking group. Overdrafts will be considered as being past due once the customer has breached an advised limit or been advised of a limit smaller than current outstandings." (See Basel Committee on Banking Supervision (2006))
[9] The data has been seasonally adjusted to avoid any potential influence from the provisioning practices of the banks.

turnover is over € 0.75 mln. The population contains more than 170,000 firms on average each year. The sample is very representative of the French businesses population and of the SMEs population in particular.

The German data have been provided by a significant proportion of both small and large German banks. This allows us to consider even the smallest borrowers in terms of turnover and amount owed. The data set is representative for the whole universe of German corporate lending, while minimizing the risk of any potential double-counting. Consistent with the aim of our study, the data set also contains information on the amount owed (as defined in Art. 501 CRR) by the borrowers in the sample and extends the data set used in Düllmann and Koziol (2014). The sample includes a total of over 5.4 million observations. Credit quality is measured in terms of IRB PDs which are mapped to a consistent master scale[10]. Although the vast majority of banks in the sample have adopted the RSA, their rating system has been designed along the requirements of an IRB rating system.

As zero defaults were observed for certain points in time for both data sets, some rating classes had to be merged to ensure the robustness of the estimations. For Germany we obtain five rating classes (from six in the master scale). For France the original rating scale of ten grades is collapsed to four rating classes for the purposes of this study. Figures 2 and 3 depict the time series of defaults rates for the different rating or turnover categories in both countries. Low default rates are observed for borrowers of high credit quality rating and larger corporates classes. Lower credit quality rating and smaller SMEs classes are related to higher default rates.

[10] The master scale was set up by the Joint Banking Initiative for the Financial Location of Germany (IFD).

Figure 2: Default Rates over time (all size classes, by rating)

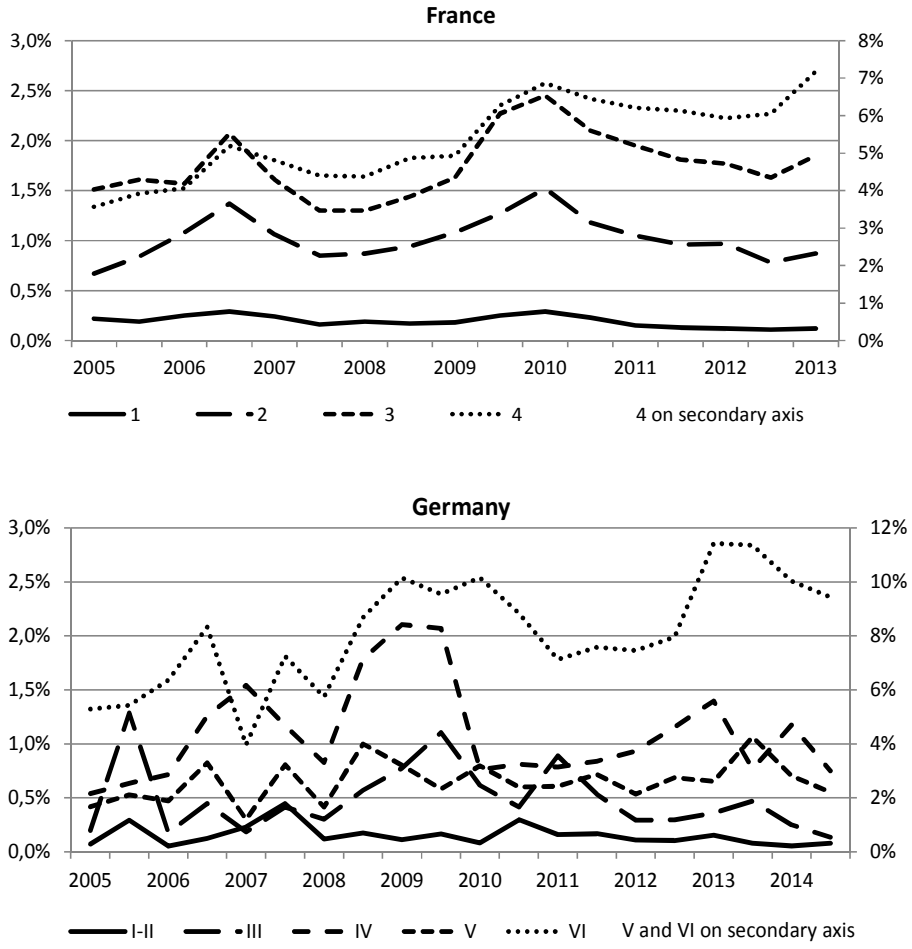
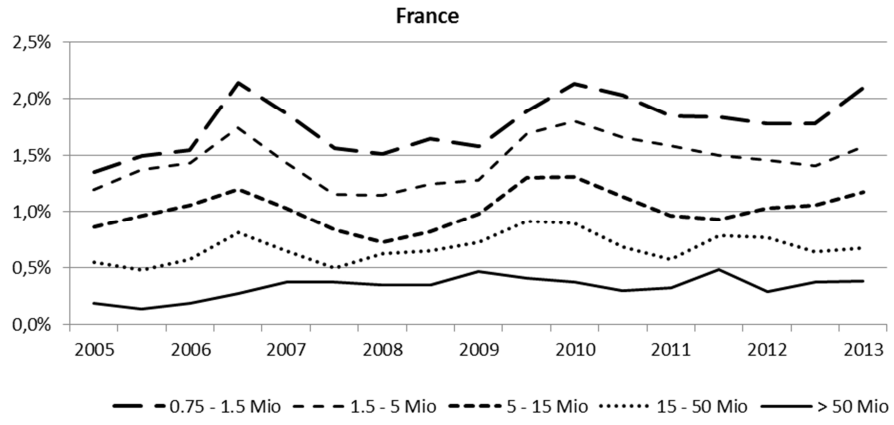


Figure 2 shows the history of default rates for the years 2005 to 2014. The upper panel depicts the default rates for rating categories 1 to 4 for France. The lower panel depicts the default rates for rating categories I-VI for Germany.

To differentiate between large corporates and SMEs and to explore a potential size dependence of asset correlation we cluster firms according to their annual turnover. We follow Art. 501 CRR which uses the turnover criterion of Article 2 of Recommendation 2003/361/EG as the single criterion to identify SMEs and define the benchmark group large corporates as firms with an annual turnover above € 50 mln. The rich population of SMEs in both samples allows to distinguish additional SME size classes. Figure 1 shows the time series of default rates for different firm size buckets. For France there are indications of a size dependence of default rates. At each point in time the default rates are negatively related to firm size, i.e. the default rates are generally lower for larger turnover classes. The picture for Germany is less straightforward, as there is no clear indication of a size dependence that is

consistent over time. On the contrary, it can be inferred that some of the highest and some of the lowest default rates observed over time can be allocated to larger turnover classes.
To assess the impact of the SME SF on regulatory capital requirements, both samples contain information on the outstanding loan volume (obligo). As the data have been extracted from the French credit register, only borrowers with a minimum obligo of €25,000 are covered. This total obligo includes not only funds effectively granted to the firm (or drawn credit), but also the bank's commitments on credit lines (or undrawn credits) and guarantees, as well as specific operations (medium and long-term lease with purchase option, factoring, securitized loans, etc.). For Germany, the amount owed as defined in Article 501 CRR is used. The structural differences between the French and the German SME sectors are mirrored in the descriptive statistics presented in Table 2, which depicts the percentage of SME SF eligible loans in each turnover class. In total, 86% of all loans to French SMEs and 64% of all loans to German SMEs are eligible for the application of the SME SF. The percentage of SME SF-eligible loans decreases with firm size. The SME SF may be applied to nearly all of the French loans and roughly seven out of ten loans to German firms in the smallest size class. In contrast, only half of the loans to German or French medium-sized corporates with a turnover just below € 50 mln are eligible to benefit from the SME SF. In contrast with Germany, the distribution of loans in France is very skewed, as the 14% of loans not benefiting from the SF represent around two-third of the total outstanding loans in terms of exposure.

Figure 3: Default Rates over time (all ratings, by size)



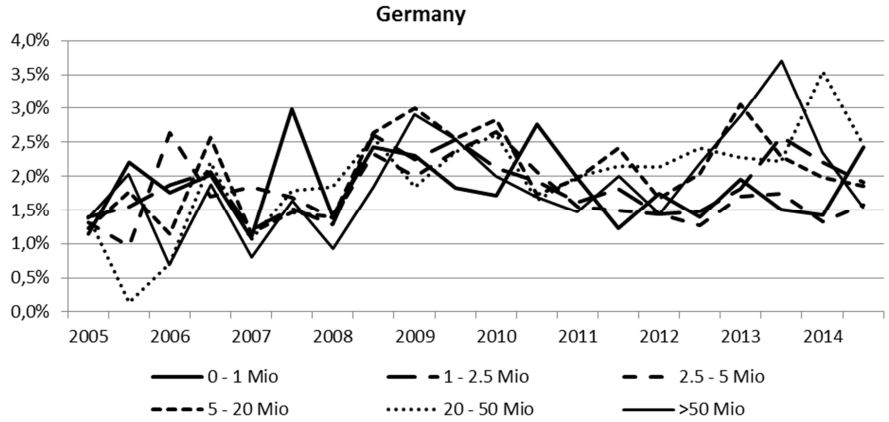


Figure 3 shows the history of default rates for the years 2005 to 2014 for different size classes for France and Germany, respectively.

Table 1: Overview applied data bases for Germany and France

Country	France	Germany
Sources	French Credit Register and Banque de France (BdF) rating system	Data provided by significant proportion of German banks. Use of IRBA ratings mapped to a consistent master scale
Time period	Q4-2004 to Q4-2013 (20 observations)	Jan 2005 to Dec 2014 (20 observations)
Data frequency	Quarterly aggregated to semi-annual	Semi-annual
Credit exposure amount	>EUR 25 000	All; measured in terms of "amount owed" as defined in Art.501 CRR
Default definition	Two criteria: legal failure (bankruptcy) and bank default, which corresponds to severe banking problems	Basel II/III default definitions
Firm's size classes definition	Restricted to firms with turnover over € 0.75 mln; Five size classes turnover measured in € mln: 0.75 to 1.5, 1.5 to7.5, 7.5 to 15, 15 to 50, And over 50.	Six size categories turnover measured in € mln: [0;0.1], (1;1.25], (2.5; 5], (5;20], (20;5], And over 50.
Number of rating grades	4, from 10 in the master scale	5, from 6 in the master scale

Table 1: SME loans eligible for SME Supporting Factor in relation to total loans (in percent)

France	Turnover in € mln	Retail	Corporate			
		0,75 - 1,5	1,5 - 5	5 - 15	15 - 50	all
	% of loans	96%	90%	67%	44%	86%

Germany	Turnover in € mln	Retail		Corporate			
		0 - 1	1 - 2.5	2.5 - 5	5 - 20	20 - 50	all
	% of loans	69%	68%	63%	55%	45%	64%

Table3: Mean weights for ratings per Turnover class

France	Turnover in € mln	Retail	Corporate			
		0,75 - 1,5	1,5 - 5	5 - 15	15 - 50	BM
	Low Risk 3	39.2%	49.4%	54.8%	56.2%	63.8%
	4	21.3%	20.1%	20.1%	20.7%	17.6%
	5	21.0%	16.2%	12.3%	11.8%	9.6%
	High Risk 6	18.6%	14.3%	12.8%	11.3%	9.0%

Germany	Turnover in € mln	Retail		Corporate			
		0 - 1	1 - 2.5	2.5 - 5	5 - 20	20 - 50	BM
	Low Risk I-II	33.7%	40.4%	44.9%	48.9%	56.2%	58.5%
	III	21.5%	22.9%	18.7%	18.4%	20.0%	19.7%
	IV	17.4%	17.3%	16.1%	15.9%	12.7%	13.6%
	V	11.9%	10.4%	10.6%	9.8%	6.8%	6.0%
	High Risk VI	15.5%	9.0%	9.6%	7.0%	4.4%	2.2%

4. Estimation methodologies and relative calibration

In this section, we first present the conceptual framework that we use, which is the structural single-factor model devised by Merton (1974). This widely known ASRF model is also the foundation of the IRB risk weight functions for credit exposures in the banking book (Gordy, 2003). Second, we specify this model as a GLMM to produce estimates of the asset correlation parameters. Third, we present our "relative calibration" methodology where large corporates IRB risk weights serve as benchmarks.

4.1 The asymptotic credit risk framework

The ASRF model belongs to the class of structural credit risk models devised by Merton (1974). In this approach, losses at the portfolio level can be defined as the sum of individual losses on defaulting loans in the portfolio, adjusted for the severity of individual losses. In other words, portfolio-level losses may be regarded as the sum of the losses given default for each individual loan in the portfolio that goes unpaid. Thus, if u_i is defined as the loss given default (LGD) of an obligor i and if $\mathbf{1}_{D_i}$ is defined as the default indicator variable of obligor i, then the total portfolio losses PL may be defined as follows:

$$PL = \sum_{i=1}^{n} u_i \mathbf{1}_{D_i}$$

In structural credit risk models, default occurs if the value of an obligor's assets is smaller than the value of the obligor's debt that is due. Because asset and debt values may be difficult to observe, this framework has been extended by generalizing the modeling of default as the crossing of an unobservable threshold.

Thus, default is triggered in this model if the ability-to-pay process Y_i of firm i falls below an exogenous default threshold γ_i. Y_i follows a standard normal distribution. It can be decomposed into the return of a systematic and unobservable factor X and an idiosyncratic firm-specific part ε_i :

$$Y_i = \sqrt{\rho_i}\ X + \sqrt{1-\rho_i}\ \varepsilon_i$$

X and ε_i are independent for every obligor i and follow a Gaussian distribution. The factor loading $\sqrt{\rho_i}$ of the systematic risk factor can be interpreted either as the sensitivity against systematic risk or as the square root of the asset correlation ρ_i. For this analysis the common assumption of a constant ρ_i is applied. The Bernoulli variable L_i describes if a credit event has occurred during the considered horizon (L_i = 1) or not (L_i = 0). It is important to differentiate between the unconditional and the conditional default probability. The unconditional default probability of obligor i for the time period t is defined as follows:

$$P(L_i = 1) = P(Y_i < \gamma_i) = \Phi(\gamma_i)$$

where Φ denotes the cumulative distribution function of a standard normal distribution.

The implementation of the single model requires to specify the dependence structure of the risk factor and to estimate the default thresholds and factor sensitivities. When using a random effect specification of the risk factor, there is a correspondence between the conditional default probability and econometric approach grounding on GLMMs.

4.2 Econometric estimation of the portfolio's credit risk parameters

Thus, to estimate default thresholds and risk factor sensitivities, we use a model that belongs to the class of GLMMs. This model combines fixed and random effects for observable and (latent) unobservable factors. Detailed presentations of GLMM models in credit risk modeling can be found in Frey and McNeil (2003) and McNeil and Wendin (2007).

If, in a general case, Y is defined as the ($N \times 1$) vector of observed default data and if γ is defined as the ($K \times 1$) vector of random effects, then the conditional expected default probability of obligor i may be expressed as follows:

$$E\left[L_i \quad = 1 \middle| \gamma\right] = g(X_i\beta + Z\gamma)$$

where $g(\cdot)$ is a differentiable monotonic link function, L_i is the default indicator variable for obligor i (L_i takes a value of 1 if there is a default and equals 0 otherwise), X is a ($N \times P$) matrix that contains the (observed) fixed effects, and Z is the ($N \times K$) design matrix for the random effects. In case of a single factor specification, K is equal to one.

In the following application, we will consider the effect of a single random general factor and we will focus on the probit link function because the normal distribution is the underlying link function that is assumed by the Basel II framework of credit risk; thus, $g(x) = \Phi(x)$. The random effect is assumed to follow a standard normal distribution. In the equation above, β is the vector of parameters that is associated with fixed effects. Considering a portfolio of N obligors who are categorized into $r = 1, \ldots, R$ (non-default) rating classes and given a vector $\gamma_t$of random effects, the conditional default probability of borrower i at time t may be expressed as follows:

$$P(L_{ti} = 1|\gamma_t) = \Phi(x'_{ti}\mu_r + z'\gamma_t)$$

where μ_r denotes the vector of parameters from the fixed effect of the borrower's rating class. If the rating scale is properly built, we expect these thresholds to be ordered and increasing as credit quality decreases. In the above equation, $x'_{ti} = [0,...,1,...0]$ is a ($1 \times R$) vector of dummies that defines the rating of borrower i during time period t. Because we assume that borrowers in a given size class are interchangeable, the estimation of this vector does not involve individual

borrowers but instead uses the periodical default rates within segments. This approach leads to an assumption of borrower homogeneity for each credit rating that is examined.

For this paper, we restrict the model to one random factor and one fixed factor (the firm's rating). We assume that the general risk factor (the risk factor of the single factor model) represents the impact on default rates of variations in general economic conditions (the "state of the economy"), i.e. the systematic risk factor of the ASRF model. The corresponding coefficient specifies the key figure of our analysis, namely the asset correlation. In this specification, the linear predictor in the regression contains an intercept term that randomly varies at the year level, the highest level in the modelling, where all other effects are nested in. In other words, a random intercept is drawn separately and independently for each year. This structure implies that a given obligor is only affected by the factor representative of general economic conditions.

Systematic risk factor is a latent factor and it corresponds to the random effect.

For each run of the asset correlation estimation, a time series of default rates is applied encompassing 20 observations in the case of Germany and 18 in the case of France.

4.3 Minimum capital requirements and relative calibration

Since we are ultimately concerned with the calibration of capital requirements, we consider not only the asset correlation estimates but also capital requirements dependent on these estimates. More precisely, we consider the empirical risk weight function, i.e. the risk weight function based on the empirically estimated asset correlations $\hat{\rho}$, rather than the asset correlation estimates themselves, in order to assess the calibration of the capital requirements:

$$RW^{Est}(\hat{\rho}, PD) = 1.06 \cdot 12.5 \cdot LGD \cdot \left[\Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\hat{\rho}} \cdot x^{*}_{99.9\%}}{\sqrt{1-\hat{\rho}}}\right) - PD\right] \cdot f(M, PD)$$

where LGD denotes the Loss Given Default, $x^{*}_{99.9\%}$ the 99.9% quantile of the standard normal distribution function and f(M, PD) the maturity adjustment dependent on the effective maturity M and the PD with f(M, PD) = (1 + (M – 2.5) ·b(PD))/(1–1.5 · b(PD)) and b(PD) = (0.11852–0.05478 ·log(PD))^2. The LGD is set to 0.45 and the maturity M to 2.5 years in our analysis. These specifications leverage on the assumption for the Foundational IRB Approach. The LGD assumption is deemed to be conservative for SME loans as 0.45 (EBA, 2016).

The current Basel III capital requirements are calculated according to the IRBA formulae for corporate exposures:

$$RW^{BIII}(PD,S) = 1.06 \cdot 12.5 \cdot LGD \cdot \left[\Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho(PD,S)} \cdot x^{*}_{99.9\%}}{\sqrt{1-\rho(PD,S)}}\right) - PD\right] \cdot f(M,PD)$$

Turnovers above € 50 mln are lumped together in a single bucket, as the risk weight curve would remain flat above this turnover threshold (for a constant PD). For a turnover above € 2.5 mln, we have applied the corporate risk weight function including the capital relief due to the turnover dependence of the asset correlation:

$$\rho(PD,S) = 0.24 - (0.24 - 0.12) \cdot (1 - e^{-50PD}) \cdot \left(1 - \frac{\min\{50, \max\{S,5\}\} - 5}{45}\right)$$

with the last term of the function being the size adjustment for SMEs.

The retail risk weight curve (other retail) has been applied for a turnover below € 2.5 mln.[11] The retail risk weight curve differs from that for corporate exposures because it does not depend on the effective maturity M and size S. The corresponding asset correlation is lower than that for the corporate portfolio and ranges from 0.03 to 0.16:

$$\rho(PD) = 0.16 - (0.16 - 0.03) \cdot (1 - e^{-35PD})$$

In both cases, the capital charge is determined by multiplying the exposure at default with the risk weight and the solvability coefficient of 0.08. The risk weights in the RSA are not based on models. More precisely, they are determined by a simple step function with 100% for loans in the corporate portfolio without an external rating and 75% for loans in the retail portfolio. This construction implies that the RSA risk weights are only partially risk-sensitive. In Germany, SMEs typically do not have external ratings.

In the CRR/CRD IV, risk weights for SME loans differ from the risk weights under Basel III. The CRR has introduced in Art. 501 a reduction in capital requirements for exposures to SMEs by applying the SME SF of 0.7619 to the risk exposure amounts. This capital discount came into force in January 2014. The SME SF can be applied to all loans granted to SME borrowers which fulfil the following criteria. (1) The loan is allocated to the corporate, retail or secured by an immovable property portfolio and is non-defaulted. (2) The borrower represents an SME defined as showing turnover of below € 50 mln. (3) The total amount owed to the lending institution, its parent and subsidiary undertakings, excluding claims or contingent claims secured on residential property collateral, shall not exceed € 1.5 mln. Against this background, the effective risk weights for loans to SMEs under the CRR/CRDIV can be derived as follows:

[11] Analyses of the Bank for the Accounts of Companies Harmonized (BACH) database from the European Committee of Central Balance Sheet Data Offices support the consideration of the first three turnover classes as other retail, since the average ratio of turnover to liabilities of credit institutions amounts to 3.1 in 2009 and € 1 mln is the exposure threshold for the retail portfolio.

$$RW^{CRR}(PD,S) = 0.7619 \cdot RW^{BIII}(PD,S) \quad \textbf{\textit{if borrower is an SME}}.$$ [12]

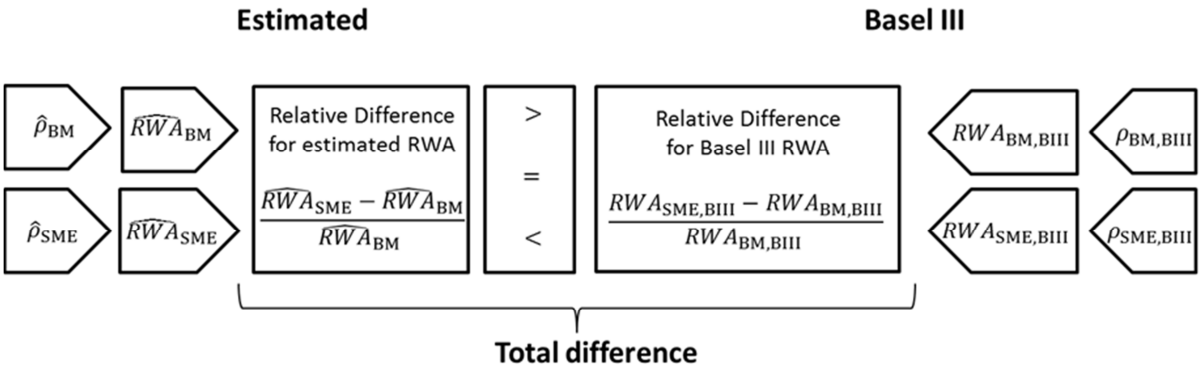
The formulas for the regulatory asset correlations remain in a similar way. The risk weights for non-SMEs, which are only large corporates in our study, are not affected by the SME SF.

The conducted analysis is very much in the spirit of previous analyses that were carried out for the relative calibration of Basel II: it explores the dependence of systematic risk on firm size and compares the size of this effect with the capital relief granted to SME lending in the regulatory minimum capital requirements of Basel III and CRR/CRD IV (Figure 3).[13] The asset correlations are estimated based on the ASRF model underlying the IRBA capital requirements. We use large corporates (that is corporates with a turnover of more than € 50 mln) as a benchmark, which means that they are assumed to be correctly calibrated in level. This is motivated by the fact that the BCBS has spent substantial effort on calibrating these portfolios due to their immense economic importance. For each size class we therefore compare the relative difference (difference in capital requirements for this size class of SMEs relative to capital requirements for the benchmark, i.e. for large corporates) of both (1) capital requirements based on estimated asset correlations and (2) the current IRBA capital requirements.

[12] For the purposes of our study we assume that all SME loans are eligible for the application of the SME SF.

[13] In this study, we consider only the relative calibration since the appropriate level of regulatory capital cannot be satisfactorily assessed for the following two reasons: 1) The overall level of capital requirements was determined in the top-down calibration of the whole Basel II framework, also involving for example the 99.9% confidence level of the value-at-risk, the scaling factor of 1.06 for credit-risk-weighted assets, and the benchmark maturity of 2.5 years. There is no reason to believe that this very different calibration goal will provide asset correlations similar to the estimates from time series of default rates. 2) Gordy and Heitfield (2010) and Düllmann et al. (2010) show that asset correlation estimates can generate significant downward biases when the underlying time series of default rates are short. Through a relative comparison of asset correlation estimates for large companies with SMEs, both of which are affected by this estimation bias, we expect to mitigate the impact of this effect.

Figure 4: Illustration of framework to compare estimated and regulatory risk weights[14]



The left-hand side of Figure 4 illustrates the estimation process of the relative difference of the SMEs' RWA to the benchmark RWA (large corporates with turnover > € 50 mln). After estimating the respective asset correlation, the RWA can be calculated. The relative difference can finally be compared to the regulatory risk weights.

5. Results

5.1 Asset correlation estimates

For the relative calibration, following Frey and McNeil (2003), we run the estimation separately for each size class. Thus, the firm's rating acts as a potential driver of the estimation of asset correlations (e.g. Hahnenstein, 2004; Düllmann and Scheule, 2006; Düllmann and Koziol, 2014), besides a general random risk factor. This two-dimensional dependence is also reflected in the current IRB risk weight functions. Therefore, we estimate the asset correlation for each turnover class taking into account all information on the different rating classes. This is an advantage in comparison to the ML estimator for which asset correlations have to be estimated separately for each rating and size bucket. The estimation results for the asset correlations using the GLMM SF estimator are presented in Table 4.

[14]BM refers to the benchmark group (i.e. large corporates, turnover larger than € 50 mln). To differentiate the risk weights calculated using the empirical asset correlations (left-hand side) from the regulatory risk weights (right-hand side) the latter are indexed BIII (for Basel III).

Table 4: GLMM SF estimates for assets correlations (in percent)[15]

	Turnover in mio €	Retail	Corporate			
		0.75 - 1.5	1.5 - 5	5 - 15	15 - 50	> 50
France	Estimates	0.56	0.59	0.62	0.69	1.36
	st. errors	(0.15)	(0.15)	(0.18)	(0.22)	(0.54)
	Likelihood Ratio test statistics	134.9***	149.9***	150.1***	161.1***	---

	Turnover in mio €	Retail		Corporate			
		0 – 1	1 - 2.5	2.5 - 5	5 - 20	20 - 50	> 50
Germany	Estimates	0.57	0.57	0.51	0.80	0.92	1.84
	st. errors	(0.19)	(0.19)	(0.18)	(0.28)	(0.36)	(0.67)
	Likelihood Ratio test statistics	40.0***	42.9***	61.3***	61.3***	44.2***	---

Table 5: Average Probability of Default at the one year horizon by size class and rating (in percent)

	Turnover in mio €	Retail	Corporate				Weighted Avg.
		0.75 - 1.5	1.5 - 5	5 - 15	15 - 50	> 50	
France	Low Risk 1	0.25%	0.22%	0.14%	0.10%	0.04%	0.20%
	2	1.07%	1.15%	0.92%	0.62%	0.33%	1.03%
	3	1.68%	2.04%	1.83%	1.12%	0.60%	1.80%
	High Risk 4	5.97%	5.64%	4.18%	3.09%	2.03%	5.38%

	Turnover in mio €	Retail		Corporate				Weighted Avg.
		0 – 1	1 - 2.5	2.5 - 5	5 - 20	20 - 50	> 50	
Germany	Low Risk I-II	0.60%	0.48%	0.48%	0.39%	0.41%	0.43%	0.50%
	III	1.57%	1.76%	1.67%	1.58%	1.76%	1.49%	1.63%
	IV	3.73%	4.27%	3.93%	3.70%	4.49%	3.78%	3.88%
	V	7.94%	10.60%	8.53%	9.07%	11.17%	10.35%	8.78%
	High Risk VI	24.23%	28.72%	25.42%	27.03%	27.07%	30.59%	25.33%

Since in the German sample the time periods in the sample cover six months we have transformed the estimates of a half-year PD_h by the formula $PD = 1 - (1 - PD_h)^2$ into PDs for a one-year horizon. This transformation is necessary for the analysis of the capital requirements since PDs in Basel III always refer to a one-year horizon. Consistent with the focus on the relative calibration based on systematic risk, the risk weights are calculated using the same

[15]In order to check the difference of the asset correlation estimates a Likelihood Ratio Test (e.g., Efron, 1967) is applied. The benchmark category (loans of borrowers with turnover > € 50 mln) is compared with each SME loan category separately. In Table 4 the Likelihood Ratio test statistics is reported and the respective significance level which is calculated based on a χ^2 distribution with two degrees of freedom. ***, **, * denote significance at the 1, 5 and 10 % level, respectively.

average PD for each rating class. Table 5 illustrates the average PDs used throughout the paper as well as the underlying PD estimations by size and rating classes using the ML estimator by Gordy and Heitfield (2010).[16]

The overall asset correlation results are consistent across Germany and France and robust for different estimators.[17] The study determines as a key finding that large corporates (Basel definition: corporates with turnover > € 50 mln) face a considerably higher systematic risk than SMEs (Figure 5), and that there is a structural difference between loans to large corporates and SME loans. More precisely, the asset correlations for large corporates are estimated to be twice as high as the asset correlations for SME loans. The systematic risk for SMEs is rather stable and does not vary significantly with turnover. For Germany, the asset correlation estimates tend to increase with firm size. This is in line with the existing academic literature which finds that asset correlations increase with firm size (e.g. DE: Düllmann/Scheule, 2006; Düllmann/Koziol, 2014; FR: Dietsch/Petey, 2004; Dietsch/Fraisse, 2013; IT: Gabbi/Vozzella, 2013; JP: Hashimoto, 2009; US: Bams/Pisa/Wolff, 2015). Irrespectively of the sample used, the level of asset correlations never exceeds two percent and is on average considerably below the asset correlations in the IRB capital requirements. A possible underestimation of the asset correlations could result from the fact that for each size class the correlations were estimated for well diversified portfolios with respect to business sectors.

[16] The estimation results are almost identical to the simple average of the default rates.

[17] In addition to the GLMM single factor estimator, the ML estimator of Gordy and Heitfield (2010) and the GLMM multi factor estimator were applied. Their estimation results are line with the estimates for the GLMM single factor. (Tables B.1 and B.2 in the Appendix)

Figure 5: Estimated asset correlation subject to firm size (turnover in mln €)

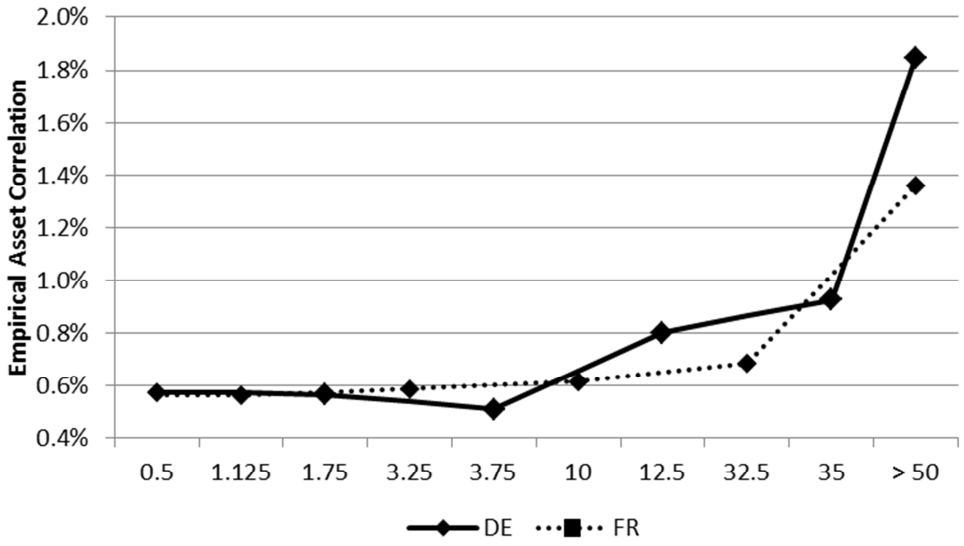


Figure 5 shows empirical asset correlations for different size classes. The x-axis depicts size classes ranging from € 0.5 mln to more than € 50 mln. The solid and the dashed lines represent asset correlations for German and French corporations, respectively.

In the next subsection we compare the capital requirements in Basel III dependent on turnover with the estimated capital requirements based on the asset correlation and PD estimates. Afterwards, we evaluate the estimated capital requirements with respect to the RSA. The risk weights are found to depend considerably on the rating/PD and on the turnover. In the following, we account for this two-dimensional dependence by weighting the IRB or RSA risk weights with respect to the number of borrowers in each rating category. The advantage of this aggregation is that we can condense the assessment of the asset correlation estimates in a single figure.

5.2 Consistency of own funds requirements

By comparing the size dependence of estimated capital requirements (i.e., based on empirical asset correlation estimates) with the size dependence hard-wired into the corresponding IRBA capital requirements, the question of whether the size dependence of IRBA capital requirements is appropriate in light of the new empirical results can be answered. In the next step, the size dependence within the RSA is investigated. For this purpose, the relative level of capital requirements implied by the asset correlation estimates are compared with the RSA capital requirements. According to Basel III, the RSA risk weight function is simply a step function with a risk weight of 100% if the firm is treated as a corporate exposure and 75% if it

is assigned to the retail portfolio, i.e. if the exposure to the borrower does not exceed € 1 mln, which is comparable with a turnover of up to € 2.5 mln.

After analyzing Basel III capital requirements, this study focusses on the capital requirements according to CRR/CRDIV including the SME SF. In this way, the impact of the SME SF can be measured when we compare the size dependence of CRR risk weights with that of the estimated risk weights. It is assumed that the SME SF is applied to all SME loans which is a rather conservative assumption given that the percentages of all loans assigned to the SME SF amount to 64% for DE and 86% for FR (Table 2).

In order to quantify the deviation of the estimated and the regulatory risk weights, the relative difference towards the benchmark of large corporates is compared by subtracting the relative difference[18] of the estimated capital requirements from the relative difference of the regulatory capital requirements. This difference, the total average difference (shown, e.g., for Basel III capital requirements in the fourth row (C-A) of Tables 5 and 6) determines whether the size dependence of Basel III or CRR/CRD IV capital requirements complies with that of estimated capital requirements. Tables 5 and 6 show the results for loans under the IRBA and the SA based on both the Basel III framework (C-A) and on the CRR/CRD IV (C-B).

For the Basel III framework, both (relative) differences are negative and the absolute value of the difference for the empirical estimates is significantly higher than that of the difference for the regulatory numbers for loans assigned to the corporate portfolio. This may be interpreted as an indication that the empirical results ceteris paribus would support lower Basel III capital requirements for SMEs. However, the gap between both relative differences from the benchmark is close to zero for loans in the IRBA retail portfolio.

The results for the SA are considerably stronger than those for the IRBA under the Basel III framework. The estimated capital requirements differ to a much greater extent from the benchmark large corporates (-37% up to -56%) than the regulatory figures (0% up to -25%). For SMEs in the corporate portfolio, the results are directionally in line with those for the IRBA, but the average total differences are higher, up to a level of 56 percentage points. In comparison to the corporate portfolio, the empirical results for the SME loans in the retail portfolio indicate a lower capital relief potential between of 19 and 28 percentage points. To

[18] The overall relative difference per size bucket is derived from the respective relative differences for each rating category by weighting with the number of loans.

sum up, for all loans assigned to the SME portfolio, the empirical results suggest that the relative reduction compared to large firms is significantly higher than reflected in the current capital requirements under the Basel III framework.

Under the CRR/CRD IV, the results in Tables 5 and 6 indicate for the IRBA that SME SF is able to compensate the difference between estimated and CRR/CRDIV capital requirements for loans in the corporate portfolio. For loans assigned to the retail portfolio the SME SF increases the size dependence even higher than the estimated risk weights suggest. The effect is almost zero for German loans, but stronger for French loans. These results are likely to overstate the additional impact of the SME SF on regulatory risk weights, as the assumption that all SME loans can be assigned to the SME SF appears to be very conservative in light of Table 2.

In the case of the CRR/CRD IV SA, the SME SF reduces the total differences between estimated and CRR/CRD IV capital requirements. For loans assigned to the corporate portfolio, the SME SF compensates some part of the total differences, but some differences still remain. For retail loans, the differences are mostly captured by the SME SF, which means that the SME SF achieves the expected purpose.

Table 5: Average total differences of capital requirements in the Basel III and CRR/CRDIV IRBA and SA for France

	Turnover (in EUR mln)			Retail	Corporate			
				0.75 - 1.5	1.5 - 5	5 - 15	15 - 50	BM
IRBA	A	Regulatory	Basel III	-54.5%	-22.1%	-19.6%	-8.7%	0.0%
	B		CRR/CRDIV	-65.3%	-40.6%	-38.7%	-30.4%	0.0%
	C	Estimated		-43.5%	-42.4%	-40.8%	-36.7%	0.0%
	C-A	Average total difference Basel III		11.0 pp	-20.3 pp	-21.2 pp	-28.0 pp	0.0 pp
	C-B	Average total difference CRR/CRDIV		21.8 pp	-1.8 pp	-2.1 pp	-6.2 pp	0.0 pp

	Turnover (in EUR mln)			Retail	Corporate			
				0.75 - 1.5	1.5 - 5	5 - 15	15 - 50	BM
SA	A	Regulatory	Basel III	-25.0%	0.0%	0.0%	0.0%	0.0%
	B		CRR/CRDIV	-42.9%	-23.8%	-23.8%	-23.8%	0.0%
	C	Estimated		-43.5%	-42.4%	-40.8%	-36.7%	0.0%
	C-A	Average total difference Basel III		-18.5 pp	-42.4 pp	-40.8 pp	-36.7 pp	0.0 pp
	C-B	Average total difference CRR/CRDIV		-0.6 pp	-18.6 pp	-17.0 pp	-12.9 pp	0.0 pp

Note: The average total difference is calculated as the difference between the regulatory and estimated relative difference in RWA. A negative difference means that the regulatory asset correlation leads to higher relative capital requirements than the risk weight based on estimated correlation (given the same level of other parameters). A positive difference means that the regulatory asset correlation leads to lower relative capital requirements than the risk weight based on estimated correlation (given the same level of other parameters).

Table 6: Average total differences of capital requirements in the Basel III and CRR/CRDIV IRBA and SA for Germany

	Turnover (in EUR mln)			Retail		Corporate			
				0 - 1	1 - 2.5	2.5 – 5	5 - 20	20 - 50	BM
IRBA	A	Regulatory	Basel III	-53.7%	-53.4%	-22.1%	-18.5%	-7.4%	0.0%
	B		CRR/CRDIV	-64.7%	-64.5%	-40.7%	-37.9%	-29.5%	0.0%
	C	Estimated		-51.8%	-52.8%	-55.8%	-42.0%	-36.9%	0.0%
	C-A	Average total difference Basel III		1.9 pp	0.6 pp	-33.6 pp	-23.5 pp	-29.5 pp	0.0 pp
	C-B	Average total difference CRR/CRDIV		12.9 pp	11.6 pp	-15.1 pp	-4.1 pp	-7.5 pp	0.0 pp

	Turnover (in EUR mln)			Retail		Corporate			
				0 - 1	1 - 2.5	2.5 - 5	5 - 20	20 - 50	BM
SA	A	Regulatory	Basel III	-25.0%	-25.0%	0.0%	0.0%	0.0%	0.0%
	B		CRR/CRDIV	-42.9%	-42.9%	-23.8%	-23.8%	-23.8%	0.0%
	C	Estimated		-51.8%	-52.8%	-55.8%	-42.0%	-36.9%	0.0%
	C-A	Average total difference Basel III		-26.8 pp	-27.8 pp	-55.8 pp	-42.0 pp	-36.9 pp	0.0 pp
	C-B	Average total difference CRR/CRDIV		-8.9 pp	-9.9 pp	-32.0 pp	-18.2 pp	-13.1 pp	0.0 pp

Note: The average total difference is calculated as the difference between the regulatory and estimated relative difference in RWA. A negative difference means that the regulatory asset correlation leads to higher relative capital requirements than the risk weight based on estimated correlation (given the same level of other parameters). A positive difference means that the regulatory asset correlation leads to lower relative capital requirements than the risk weight based on estimated correlation (given the same level of other parameters).

The results are also valid when considering each rating class separately. Table 7 shows the relative differences of the estimated and the Basel III (IRBA) risk weights for each rating category. The relative differences are negative for all loans assigned to the corporate portfolio. The differences vary slightly in each turnover class, but the overall result is clear. Against this background, the results are independent of the rating class, which means that the identified capital relief is determined for all rating classes of the borrowers.

Table7: Relative differences of IRBA capital requirements under Basel III

		Retail	Corporate			
		0,75 - 1,5	**1,5 - 5**	**5 - 15**	**15 – 50**	**BM**
France	**Low Risk 3**	13.3%	-22.6%	-23.1%	-29.3%	0.0%
	4	6.9%	-20.3%	-20.9%	-27.4%	0.0%
	5	6.8%	-18.6%	-19.3%	-26.4%	0.0%
	High Risk 6	15.5%	-14.4%	-15.5%	-24.1%	0.0%

		Retail		Corporate			
		0,1	**1, 2.5**	**2.5, 5**	**5, 20**	**20, 50**	**BM**
Germany	**Low Risk I-II**	-1.2%	-1.6%	-36.9%	-26.1%	-31.0%	0.0%
	III	-3.5%	-3.9%	-33.8%	-23.4%	-29.1%	0.0%
	IV	1.8%	1.4%	-30.0%	-20.1%	-27.1%	0.0%
	V	10.9%	10.5%	-28.4%	-18.7%	-25.8%	0.0%
	high Risk VI	9.2%	8.7%	-30.3%	-20.1%	-25.0%	0.0%

5.3 Impact of Exposure

In order to fully analyse the adequacy of the SME SF the threshold for its application need to be taken into account. Thus, the study also aims to assess whether the systematic risk of SMEs depends on the amount owed ("obligo") although there is no economic argument why exposure should determine the riskiness of a loan. The SME threshold was incorporated for regulatory purposes to capture the adequate recipients of the SME capital relief. Against this background, asset correlations are estimated with respect to the borrower's loan volume considering the different rating classes. The estimated asset correlations subject to obligo are shown in Table 8 and Figure 5. For SME loans the asset correlations are considerably below the ones for large corporate by almost two third for Germany and by almost one half for France. The asset correlations for the SME loans fluctuate around a rather low estimate of 0.6% (for DE) and 0.7% (for FR) across the obligo classes. For very large loans (>€ 2.5 mln) the systematic risk increases to some extent which is mainly driven by the size of the borrower which was already discussed in Section 5.1 (e.g. Table 4). Neither for Germany nor for France, any empirical evidence is found which supports the limit of € 1.5 mln currently implemented in Article 501 CRR. This means that the limit of € 1.5 mln for the amount owed set in the Article 501 CRR does not seem to be indicative of any change in the riskiness of firms.

Table 8: Asset Correlation with respect to Exposure (in percent)[19]

	Obligo (mln €)	**< 0.1**	**0.1 - 0.25**	**0.25 - 0.5**	**0.5 - 1**	**1 - 1.5**	**1.5 - 2**	**2 - 2.5**	**> 2.5**
FR	**Estimates**	0.82	0.80	0.68	0.51	0.85	0.58	0.47	0.93
	st. Errors	0.31	0.21	0.18	0.14	0.23	0.18	0.17	0.25
	Likelihood Ratio test statistics with respect to benchmark large corps	76.8***	104.0***	103.8***	53.8***	56.1***	24.5***	44.4***	6.5**

	Obligo (mln €)	**0 - 0.05**	**0.05 - 0.1**	**0.1 - 0.25**	**0.25 - 0.5**	**0.5 - 1**	**1 - 1.5**	**1.5 - 2.5**	**> 2.5**
DE	**Estimates**	0.55	0.56	0.61	0.51	0.56	0.50	0.58	0.76
	st. errors	0.22	0.23	0.24	0.20	0.23	0.26	0.31	0.40
	Likelihood Ratio test statistics with respect to benchmark large corps	46.3***	43.7***	39.1***	44.3***	22.5***	12.3***	14.4***	20.2***

[19] In order to check the difference of the asset correlation estimates a Likelihood Ratio Test (e.g., Efron, 1967) is applied. The benchmark category (loans of borrowers with turnover > € 50 mln) is compared with each SME loan category separately. In Table 8 the Likelihood Ratio test statistics is reported and the respective significance level which is calculated based on a χ^2 distribution with two degrees of freedom. ***, **, * denote significance at the 1, 5 and 10 % level, respectively. Furthermore, it was also checked whether the asset correlation estimates are statistically significantly differently for split samples, for instance all SME loans below and above € 1 mln obligo. As in each Likelihood Ratio test all SME loans are taken into account, each sample split turned out to obtain statistically significantly different asset correlations. However, from an economic perspective the differences in level are not material.

Figure 6: Estimated Asset Correlation subject to loan exposure (obligio in mln €)

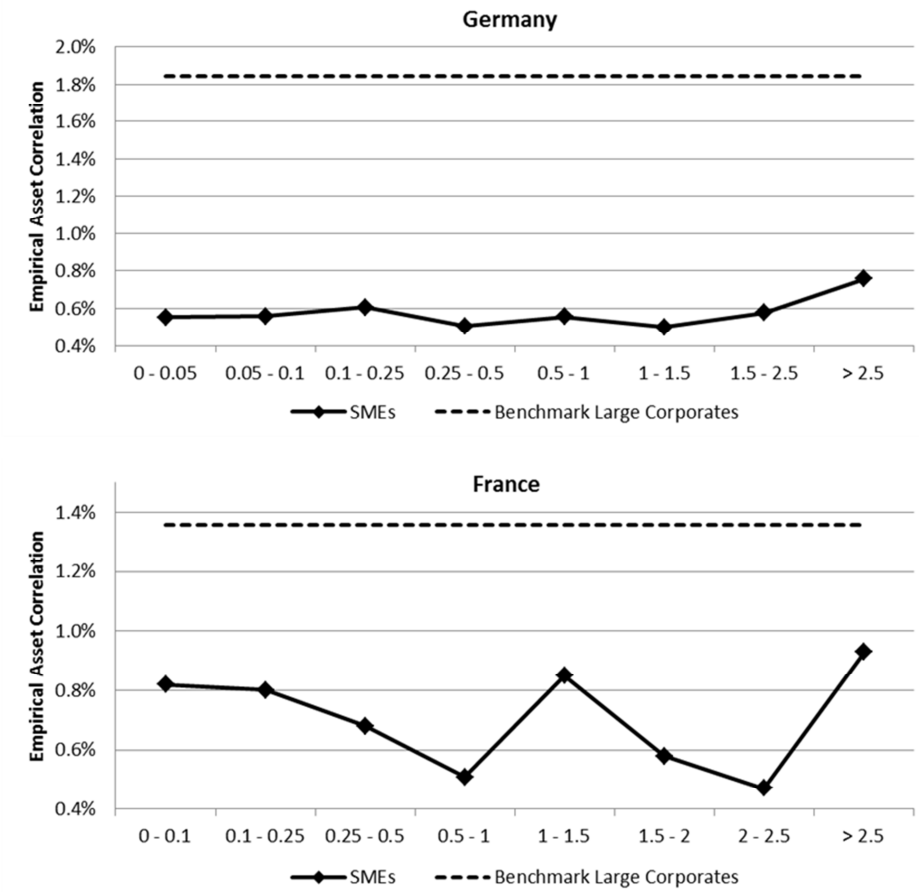


Figure 6 shows empirical asset correlations subject to the corporates' amount of loan exposure for SMEs. The x-axis depicts the loan exposure classes ranging from € 0.1 mln to more than € 2.5 mln. The upper and the lower panel show the figures for German and French corporates, respectively.

6. Conclusion and policy implications

Using comprehensive data sets covering a significant part of the German and French SME and large corporate population, this study contributes to two policy issues related to SME financing and provides significant results. The first issue concerns the relative calibration of capital requirements and the adequacy of capital requirements with the riskiness of SMEs in general. The second concerns the consistency of the SME SF introduced in the CRR/CRDIV in particular. The study addresses these issues by using the asymptotic single risk model (Gordy, 2003) and assessing firm size as a driver of systematic credit risk in this framework. Therefore, in this study, the asset correlation is used as the key measure of systematic risk.

The first policy implication concerns a potential increase of the capital relief granted for loans in the asset class "SMEs" which was supported in the Basel II/III Accord. In both the French and the German sample, results suggest that the relative differences between the capital requirements for large corporates and those for SMEs (in other words, the capital relief for SMEs) are lower in the Basel III framework than implied by empirically estimated asset correlations. For all loans assigned to the SME portfolio, the empirical results suggest that the relative reduction compared to large firms is significantly higher than reflected in the current capital requirements under the Basel III framework. On average, the asset correlations for large corporates are estimated to be twice as high as the asset correlations for SME loans. Thus, results in this paper may indicate a potential for increasing the capital relief for SMEs exposures, whatever the form of this increase: by lowering the regulatory capital requirements for SMEs, for instance by lowering the asset correlation values in the IRB formula, or by lowering the risk weights directly.

Similarly, for all loans in the RSA, the empirical results point to the potential of lowering the flat 75% retail and 100% corporate risk weights for unrated SME exposures to better reflect the size dependence that is present in the empirical risk weights. But, before drawing this inference as a policy message of this paper, the following important caveats need to be carefully considered. The RSA was deliberately calibrated more conservatively than the IRB approaches. This can be explained by the significantly lower risk sensitivity of the RSA and the regulatory intention to retain incentives in terms of a ceteris paribus capital relief when banks switch to the more risk sensitive IRB approaches. The more conservative calibration is one reason why the capital requirements in the RSA are currently independent of the firm size, which is one important driver for the empirically observed lower potential for reductions

of the capital requirements. It also suggests that at least a substantial part of the 15-35% difference between the current capital relief in the RSA and the capital relief implied by our new empirical results can be explained by this original calibration target. However, since the regulatory minimum capital requirements are internationally harmonized today, their modification appears reasonable because the results of this study cover SME lending in two major European countries and the used data set represents the largest coverage of the countries in the Eurozone.

The second policy implication concerns the SME SF. This study provides unambiguous support for the Supporting Factor, whatever the approach – IRBA or RSA – used to compute capital requirements for SME loans. This result relies on the comparison of the dependence of CRR risk weights with that of the estimated risk weights. In the IRBA, results show that SME SF in the CRR/CRDIV is able to compensate the difference between estimated and CRR/CRDIV capital requirements for loans in the corporate portfolio. For loans classified in the retail portfolio, the SME SF increases the size dependence even more than the estimated risk weights suggest. In the RSA approach, the SME SF reduces the total differences between estimated and CRR/CRD IV capital requirements. For loans assigned to the corporate portfolio, it only compensates part of the total differences, so that some differences still remain. For retail loans, the differences are mostly captured by the SME SF which means that the SME SF achieves the expected purpose.

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Appendix A: Literature Overview

Table A.1: Overview of empirical studies on the relationship between asset correlations and firm size

Study	Data	Size Dependence	Estimated Results
Lopez, 2004	US, Japan, Europe, firm credit portfolios, year-end 2000	positive	**World portfolios** SMEs [4 size classes 0-$1b]:10%-13.75%, Large corporates (>$1b): 20% **US portfolios** SMEs [4 size classes 0-$1b]:10%-22.5%, Large corporates (>$1b): 30% **Japanes portfolios** SMEs [4 size classes 0-$1b]:20%-30%, Large corporates (>$1b): 45% **European portfolios** SMEs [4 size classes 0-$1b]:11.25%-15%, **Large corporates (>$1b): 22.5%**
Dietsch/Petey, 2004	France, Germany, Coface, Credit Reform, 1995-2001, 1997-2001	negative within SMEs, but SMEs depict lower overall asset correlation than large corporates	**France** SMEs [3 groups 0-40M€]:1.54%-0.49% Large corporates (>40M€): 2.21% **Germany** SMEs [3 groups 0-40M€]:1.23%-0.14% Large corporates: (>40M€): 1.45%
Düllmann/Scheule, 2006	Germany, Credit history of German firms, 1991-2000 (Calculation of PD); 1987-2000 (Data set)	positive	**AMM estimator** Small [Rating A-C]:1.2%-3.2% Medium [Rating A-C]: 1.6%-3.3% Large [Rating A-C]:2.1% -14% **ML estimator** Small [Rating A-C]:0.9%-2.5% Medium [Rating A-C]:1.2%-5.7% Large [Rating A-C] 0.01.6%-9.4%
Hashimoto, 2009	Japan, Teikoku Databank Matrix (Default Data for Japan. companies), 1985-2005	positive	Small (Capital < 100Mil Yen): 1.5% Large + Med. S. (Capital > 100Mil Yen): 4.5%
Lee et al., 2011	US, COMPUSTAT+ (CRSP), 1988–2007	positive	**2007** Small [high-low PD]:3.40%-8.97% Medium [high-low PD]:9.38%-21.94% Big [high-low PD]: 10.26%-27.55%
Lee et al., 2012	UK, Datastream - accounting data and return index, 2000-2009	positive	**Average across years** Small [3 classes according to PD]:1.2%-3.1% Medium [3 classes according to PD]:2.5%-6.1% Big [3 classes according to PD]: 9.5%-33.2%
Haddad, 2013	Canada, SME Credit Data, 1997-2010	no relationship (size measured in terms of amount borrowed)	**AC by amount borrowed** Small and med. size borrowing [100.000-$1Mil]: 0.34%-0.60% Large borrowing [>$1Mil]: 0.68%
Gabbi/Vozzella, 2013	Italy, AIDA (Accounting data for SMEs), 1994-2008	positive (J-shaped) for total sample; several sub-samples (risk classes) show a sudden decrease for the largest size class	Total sample [5 size classes based on sales]: 6.47%-11.72% 6 sub-samples (risk classes) [5 size classes]: 4.57%-71.48%
Bams et al., 2013	US, small businesses, 2005-2011	asset correlation estimates (multi-factor) for small business very low as compared to asset correlation estimates for corporates	SMEs [10 industries & 10 rating classes] 0.0%-3.39% Corporates [15 rating classes]: 3.83%-22.18%
Düllmann/Koziol, 2014	Germany, SME and large corporate lending, 2005-2011	positive	**ML estimator** SMEs with turnover <0.3 Mil€ [3 rating classes]: 0.50%-0.56% SMEs with turnover 0.3-1 Mil€ [3 rating classes]: 0.31%-0.59% SMEs with turnover 1-2.5 Mil€ [3 rating classes]: 0.49%-0.62% SMEs with turnover 2.5-5 Mil€ [3 rating classes]: 0.64%-0.74% SMEs with turnover 5-50 Mil€ [3 rating classes]: 0.70%-0.81% Large corp.with turnover >50 Mil€ [3 rating classes]: 1.54%-1.72% **MM estimator** SMEs with turnover <0.3 Mil€ [3 rating classes]: 0.57%-0.69% SMEs with turnover 0.3-1 Mil€ [3 rating classes]: 0.36%-0.78% SMEs with turnover 1-2.5 Mil€ [3 rating classes]: 0.60%-0.91% SMEs with turnover 2.5-5 Mil€ [3 rating classes]: 0.82%-1.14% SMEs with turnover 5-50 Mil€ [3 rating classes]: 1.00%-1.09% Large corp.with turnover >50 Mil€ [3 rating classes]: 2.10%-2.45%

Appendix B: Robustness checks for asset correlation estimations

Table B.1: ML estimates for assets correlations (in percent)

	Turnover in mio €	Retail	Corporate			
		0.75 - 1.5	1.5 - 5	5 - 15	15 - 50	> 50
France	**Low Risk 1**	1.41	1.22	0.69	0.57	3.16
	p-value	(0.00)	(0.00)	(0.08)	(0.22)	(0.17)
	2	0.72	0.70	0.80	0.64	1.99
	p-value	(0.00)	(0.00)	(0.02)	(0.10)	(0.15)
	3	0.61	0.60	1.59	1.18	6.26
	p-value	(0.00)	(0.00)	(0.01)	(0.02)	(0.03)
	High Risk 4	0.79	0.94	0.71	0.94	3.02
	p-value	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)

	Turnover in mio €	Retail		Corporate		
		0 - 1	1 - 2.5	2.5 - 5	5 - 50	> 50
Germany	**Low Risk I-III**	0.85	0.68	0.75	0.61	1.79
	p-value	(0.01)	(0.01)	(0.03)	(0.02)	(0.02)
	IV	0.58	0.74	0.52	0.53	2.10
	p-value	(0.01)	(0.02)	(0.06)	(0.03)	(0.04)
	High Risk V-VI	0.47	0.42	0.42	0.85	1.93
	p-value	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)

Table B.2: GLMM multi-factor estimates for assets correlations (in percent)

	Var-Cov-Matrix for Turnover Buckets (in mio €)		Retail	Corporate			
			0,75 - 1,5	1,5 - 5	5 - 15	15 - 50	> 50
France	Retail	**0,75 - 1,5**	**2.2%**	2.4%	0.8%	-0.9%	-3.3%
	Corporate	**1,5 - 5**	2.4%	**2.7%**	1.0%	-1.1%	-3.9%
		5 - 15	0.8%	1.0%	**0.7%**	0.4%	-0.3%
		15 - 50	-0.9%	-1.1%	0.4%	**2.1%**	4.4%
		> 50	-3.3%	-3.9%	-0.3%	4.4%	**10.3%**

	Var-Cov-Matrix for Turnover Buckets (in mio €)		Retail		Corporate			
			0 - 1	1 - 2.5	2.5 - 5	5 - 20	20 - 50	>50
Germany	Retail	**0 - 1**	**0.6%**	0.4%	0.4%	0.4%	0.2%	0.5%
		1 - 2.5	0.4%	**1.0%**	0.5%	0.5%	0.6%	0.7%
	Corporate	**2.5 - 5**	0.4%	0.5%	**0.5%**	0.6%	0.5%	0.7%
		5 - 20	0.4%	0.5%	0.6%	**0.7%**	0.8%	1.0%
		20 - 50	0.2%	0.6%	0.5%	0.8%	**1.0%**	1.2%
		>50	0.5%	0.7%	0.7%	1.0%	1.2%	**1.7%**

Appendix C: Detailed results for calculation of average total differences

Table C.1: Capital requirements in terms of risk weights per rating class (in percent) using the empirical framework

	Turnover in mio €	Retail	Corporate			
		0.75 - 1.5	1.5 - 5	5 - 15	15 - 50	> 50
France	Low Risk 1	1.7%	1.7%	1.8%	1.9%	3.1%
	2	6.0%	6.2%	6.4%	6.8%	10.6%
	3	9.2%	9.4%	9.7%	10.3%	16.0%
	High Risk 4	20.1%	20.6%	21.2%	22.6%	34.1%

	Turnover in mio €	Retail		Corporate			
		0 - 1	1 - 2.5	2.5 - 5	5 - 20	20 - 50	> 50
Germany	Low Risk I-II	3.5%	3.5%	3.3%	4.4%	4.8%	7.8%
	III	8.6%	8.5%	8.0%	10.6%	11.6%	18.2%
	IV	16.2%	16.1%	15.1%	19.8%	21.5%	33.3%
	V	28.2%	28.0%	26.4%	34.2%	37.1%	56.0%
	High Risk VI	50.5%	50.1%	47.3%	60.3%	65.0%	94.3%

Table C.2: Capital requirements in terms of risk weights per rating class (in percent) using the Basel II/III framework

	Turnover in mio €	Retail	Corporate			
		0.75 - 1.5	1.5 - 5	5 - 15	15 - 50	> 50
France	Low Risk 1	19.1%	36.3%	37.3%	42.1%	46.0%
	2	49.1%	77.4%	79.8%	90.4%	98.8%
	3	59.8%	91.4%	94.3%	107.6%	118.1%
	High Risk 4	70.9%	121.9%	126.6%	147.1%	162.8%

	Turnover in mio €	Retail		Corporate			
		0 - 1	1 - 2.5	2.5 - 5	5 - 20	20 - 50	> 50
Germany	Low Risk I-II	34.5%	34.5%	58.4%	61.0%	68.7%	74.1%
	III	58.0%	58.0%	89.0%	93.2%	106.0%	114.7%
	IV	68.7%	68.7%	110.4%	116.5%	134.6%	146.6%
	V	77.1%	77.1%	147.1%	155.4%	179.5%	195.0%
	High Risk VI	116.1%	116.1%	210.6%	219.9%	245.9%	261.7%

Appendix D: Estimated and Basel III Risk Weights and Relative Differences in Capital Requirements in case of Germany

	Estimated						Risk Weights	Basel III					
	Retail		Corporate					Retail		Corporate			
	0,1	1, 2.5	2.5, 5	5, 20	20, 50	> 50		0,1	1, 2.5	2.5, 5	5, 20	20, 50	> 50
Low Risk I-II	3.5%	3.5%	3.3%	4.4%	4.8%	7.8%		34.5%	34.5%	58.4%	61.0%	68.7%	74.1%
III	8.6%	8.5%	8.0%	10.6%	11.6%	18.2%		58.0%	58.0%	89.0%	93.2%	106.0%	114.7%
IV	16.2%	16.1%	15.1%	19.8%	21.5%	33.3%		68.7%	68.7%	110.4%	116.5%	134.6%	146.6%
V	28.2%	28.0%	26.4%	34.2%	37.1%	56.0%		77.1%	77.1%	147.1%	155.4%	179.5%	195.0%
High Risk VI	50.5%	50.1%	47.3%	60.3%	65.0%	94.3%		116.1%	116.1%	210.6%	219.9%	245.9%	261.7%

	Estimated						Relative Differences in Capital Requirements by Rating and Turnover class	Basel III					
	Retail		Corporate					Retail		Corporate			
	0,1	1, 2.5	2.5, 5	5, 20	20, 50	> 50		0,1	1, 2.5	2.5, 5	5, 20	20, 50	> 50
Low Risk I-II	-54.7%	-55.1%	-58.0%	-43.8%	-38.2%	0.0%		-53.5%	-53.5%	-21.1%	-17.7%	-7.2%	0.0%
III	-52.9%	-53.3%	-56.2%	-42.1%	-36.7%	0.0%		-49.4%	-49.4%	-22.4%	-18.7%	-7.6%	0.0%
IV	-51.3%	-51.8%	-54.7%	-40.6%	-35.3%	0.0%		-53.1%	-53.1%	-24.7%	-20.5%	-8.2%	0.0%
V	-49.6%	-50.0%	-52.9%	-39.0%	-33.7%	0.0%		-60.5%	-60.5%	-24.5%	-20.3%	-7.9%	0.0%
High Risk VI	-46.5%	-46.9%	-49.8%	-36.1%	-31.1%	0.0%		-55.6%	-55.6%	-19.5%	-16.0%	-6.0%	0.0%

Total Differences of Capital Requirements in BASEL III						
	Retail		Corporate			
	0,1	1, 2.5	2.5, 5	5, 20	20, 50	> 50
Low Risk I-II	-1.2%	-1.6%	-36.9%	-26.1%	-31.0%	0.0%
III	-3.5%	-3.9%	-33.8%	-23.4%	-29.1%	0.0%
IV	1.8%	1.4%	-30.0%	-20.1%	-27.1%	0.0%
V	10.9%	10.5%	-28.4%	-18.7%	-25.8%	0.0%
High Risk VI	9.2%	8.7%	-30.3%	-20.1%	-25.0%	0.0%

Estimated							Basel III					
Retail		Corporate					Retail		Corporate			
0,1	1, 2.5	2.5, 5	5, 20	20, 50	> 50		0,1	1, 2.5	2.5, 5	5, 20	20, 50	> 50
-51.8%	-52.8%	-55.8%	-42.0%	-36.9%	0.0%		-53.7%	-53.4%	-22.1%	-18.5%	-7.4%	0.0%

Average total difference					
Retail		Corporate			
0,1	1, 2.5	2.5, 5	5, 20	20, 50	> 50
1.9%	**0.6%**	**-33.6%**	**-23.5%**	**-29.5%**	**0.0%**

Appendix E: Estimated and Basel III Risk Weights and Relative Differences in Capital Requirements in case of France

	Estimated					Risk Weights	Basel III				
	Retail	Corporate					Retail	Corporate			
	0,75 - 1,5	1,5 - 5	5 - 15	15 - 50	> 50		0,75 - 1,5	1,5 - 5	5 - 15	15 - 50	> 50
Low Risk 3	1.7%	1.7%	1.8%	1.9%	3.1%		19.1%	36.3%	37.3%	42.1%	46.0%
4	6.0%	6.2%	6.4%	6.8%	10.6%		49.1%	77.4%	79.8%	90.4%	98.8%
5	9.2%	9.4%	9.7%	10.3%	16.0%		59.8%	91.4%	94.3%	107.6%	118.1%
High Risk 6	20.1%	20.6%	21.2%	22.6%	34.1%		70.9%	121.9%	126.6%	147.1%	162.8%

	Estimated					Relative Differences in Capital Requirements by Rating and Turnover class	Basel III				
	Retail	Corporate					Retail	Corporate			
	0,75 - 1,5	1,5 - 5	5 - 15	15 - 50	> 50		0,75 - 1,5	1,5 - 5	5 - 15	15 - 50	> 50
Low Risk 3	-45.3%	-43.8%	-42.1%	-37.8%	0.0%		-58.6%	-21.2%	-18.9%	-8.5%	0.0%
4	-43.4%	-42.0%	-40.2%	-36.0%	0.0%		-50.3%	-21.6%	-19.3%	-8.6%	0.0%
5	-42.6%	-41.2%	-39.5%	-35.3%	0.0%		-49.4%	-22.6%	-20.2%	-8.9%	0.0%
High Risk 6	-40.9%	-39.5%	-37.8%	-33.7%	0.0%		-56.5%	-25.1%	-22.3%	-9.7%	0.0%

	Total Differences of Capital Requirements in BASEL III				
	Retail	Corporate			
	0,75 - 1,5	1,5 - 5	5 - 15	15 - 50	> 50
Low Risk 3	13.3%	-22.6%	-23.1%	-29.3%	0.0%
4	6.9%	-20.3%	-20.9%	-27.4%	0.0%
5	6.8%	-18.6%	-19.3%	-26.4%	0.0%
High Risk 6	15.5%	-14.4%	-15.5%	-24.1%	0.0%

Estimated				
Retail	Corporate			
0,75 - 1,5	1,5 - 5	5 - 15	15 - 50	> 50
-43.5%	-42.4%	-40.8%	-36.7%	0.0%

Basel III				
Retail	Corporate			
0,75 - 1,5	1,5 - 5	5 - 15	15 - 50	> 50
-54.5%	-22.1%	-19.6%	-8.7%	0.0%

Average total difference				
Retail	Corporate			
0,75 - 1,5	1,5 - 5	5 - 15	15 - 50	> 50
11.0%	**-20.3%**	**-21.2%**	**-28.0%**	**0.0%**